How Things Break

Solids fail through the propagation of cracks, whose speed is controlled by instabilities at the smallest scales.

Michael Marder and Jay Fineberg

Galileo Galilei was almost seventy years old, his life nearly shattered by a trial for heresy before the Inquisition, when he retired in 1633 to his villa near Florence to construct the *Dialogues Concerning Two New Sciences*. His first science was the study of the forces that hold objects together and the conditions that cause them to fall apart—the dialogue taking place in a shipyard, triggered by observations of craftsmen building the Venetian fleet. His second science concerned local motions—laws governing the movement of projectiles.

The two subjects Galileo founded have fared differently over the centuries. One has become a respectable branch of mechanical engineering, while the other has become a core subject that physicists learn at the beginning of their education. Although now, as in Galileo's time, shipbuilders need good answers to questions about the strength of materials, the subject has never yielded easily to basic analysis. Galileo identified the main difficulty: "One cannot reason from the small to the large, because many mechanical devices succeed on a small scale that cannot exist in great size."[1] Nearly three hundred years elapsed after Galileo wrote these lines before science reached the atomic scale and began to answer the questions he had posed on the origins of strength and the relation between large and small.

Despite the tremendous development of solid-state physics in this century, physicists have paid slight attention to how things break. In part, this neglect has occurred because the subject seems too hard. Cracks form at the atomic scale, extend to the macroscopic level, are irreversible and travel far from equilibrium. Many of the tools with which solid-state physics was built do not work. For example, there is no perfect lattice left in which to calculate the quantum mechanical motion of electrons, and cracks move so quickly that even basic quantities such as temperature are ill defined near their tips. There is also the embarrassment of explaining to colleagues that one is working on failure. The strength of solids calculated from an excessively idealized starting point comes out completely wrong; it is not determined by performance under ideal circumstances, but instead by the survival of the most vulnerable spot under the most adverse of conditions.

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Failure of perfect solids

Here is how a perfect solid would break. Take a block of material, of height h, and cross-sectional area A, pulled by a force F. (See figure 1.) The block separates into halves when its atoms are pulled beyond the breaking point. To estimate the force, F_B, needed to reach this goal, recall that Young's modulus, Y, relates the stress, Σ, on a body to its extension, δh, through the relation

$$\Sigma = \frac{F}{A} = \frac{\delta h}{h} Y \qquad (1).$$

Suppose that the block snaps when the atoms move apart by 20% of their original spacing; the critical stress Σ_c to make this happen is

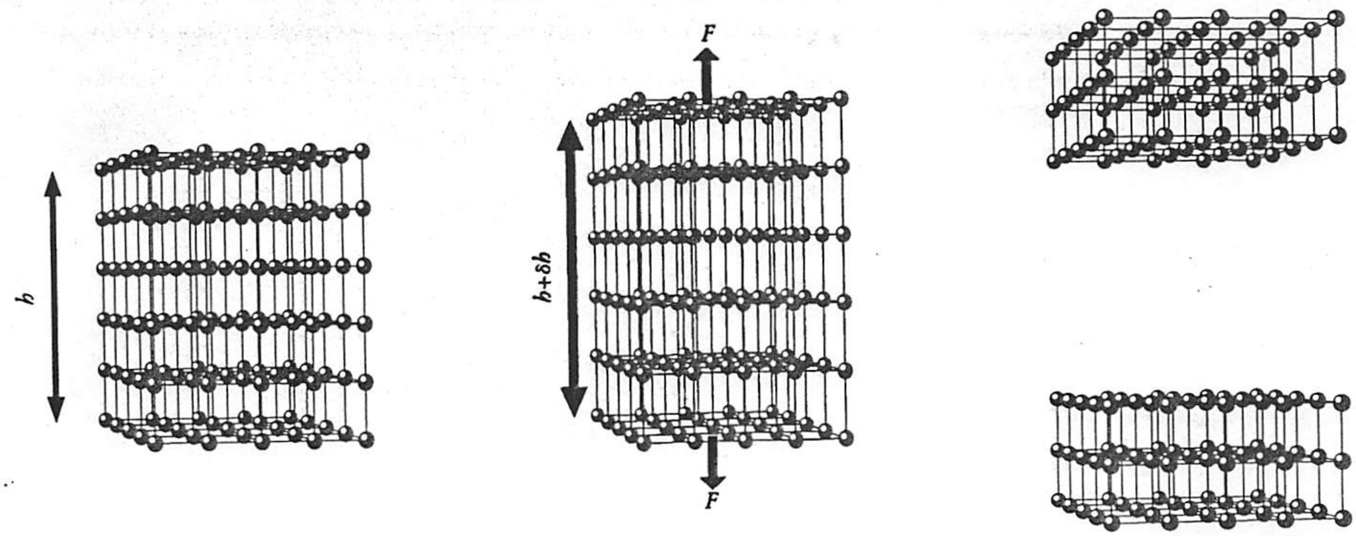
$$\Sigma_c = Y/5 \qquad (2).$$

A glance at the table on page 25 shows that the theoretical strength as estimated in equation 2 is about two orders of magnitude larger than the practical strength of a material. Although it is natural to dismiss this discrepancy as resulting from the crude approximations used to obtain equation 2, enough effort has been put into carrying out much more sophisticated quantum mechanical versions of the calculations to show that the estimate is really quite good and that the error lies elsewhere.

An engineer and a physicist compete to find the best material to build a house. The engineer chooses brick because she knows it is what everyone else uses. The physicist decides to conduct some basic research. Turning to the periodic table, he finds the element with the highest bonding strength and melting point, and first proposes diamond. Trying to find something cheaper, he next proposes a vitreous mixture of silicon and oxygen, since the raw materials are abundant and safe and form strong bonds. All is well until someone throws the first stone. In fact, the relation between bonding energies and strength of materials is far from direct; physicists had best respect the practical experience of engineers until they can really explain why one should not build glass houses.

Introduction of cracks

Flaws in materials determine strength, so it is necessary to move from an ideal material to one in which a flaw occupies the center of attention. This task was first



A FLAWLESS SOLID, initially of length h, increases in length by an amount δh, which is proportional to the force, F, applied to it. When δh exceeds a critical value, the solid breaks as a single unit into two equal pieces. However, only carefully prepared fibers of glass or certain metals have ever been made to fail in this way.[2] FIGURE 1

carried out in 1913 by G. E. Inglis. He considered a large plate of elastic material with an elliptical hole. Pulling upon the plate with a uniform stress Σ far from the hole, he found that stresses near the narrow end of the hole were much larger than Σ—by a factor $2(l/\rho)^{1/2}$, where l is the length of the hole and ρ is its radius of curvature. Just as a lightning rod generates huge electric fields, so a slit creates enormous tensions near its tip. If a flaw is sufficiently thin, it need not be particularly long to pose a threat to the body in which it lives. According to the table, brittle materials fail at stresses one hundred times smaller than one at first expects. Suppose, as A. A. Griffith did in 1921, that the materials are plagued with slits, whose tips reach a destructive stress while the rest of the body remains safely below it. Taking $\rho = 1$ angstrom, and $l = 1$ micrometer gives $(l/\rho)^{1/2} \approx 100$. This argument explains the practical strength of brittle solids, since it is quite a challenge to prepare materials without micrometer-sized flaws at the surface, ready to spring into action at stresses smaller than expected.[2,3] Notice that there is no requirement of a critical density of flaws. A single one will do. Therefore, for structures of great importance such as airplanes or nuclear containment vessels, arguments based upon the statistical likelihood of flaws are unable to guarantee safety, and case-by-case examination of the structures is essential. In addition, structures must be designed with special care to avoid making growth of flaws more likely. (See the box on page 26.)

Brittle and ductile materials

Many of the greatest successes of solid-state theory have flowed from explaining qualitative properties of solids. Why are some materials conductors and others insulators? Electron band theory provides an answer. Why are some transparent and others opaque? Calculations of the interaction of matter with light show why. The most important qualitative fact in the mechanical properties of solids is that some are brittle and shatter in response to a blow, while others are ductile, and the blow merely causes them to deform. Why?

This question is nothing but the question—in a new guise—of what makes a crack grow. Take a slab of material, make a saw cut in it, and pull. In a brittle material, the tip of the saw cut spontaneously sharpens down to atomic dimensions, and like a knife blade one atom wide, it slices its way forward.[3] In a ductile material the tip of the saw cut blunts, broadens and flows, so that great effort is required to make it progress.

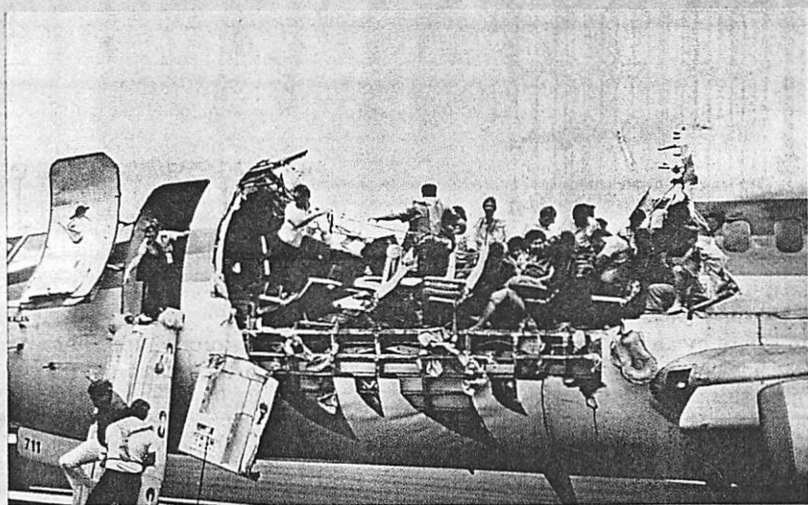
There is no completely satisfactory answer to the question of why some materials are brittle and others are ductile, as the manufacturers of atoms seem to omit this property when writing down their technical specifications. The most well developed attack on the problem considers stationary, atomically sharp cracks in otherwise perfect crystals, and asks what happens when slowly increasing stresses are inflicted upon them. In 1974, James Rice

The practical and theoretical strengths of materials.

Material	Young's Modulus Y (10^{11}dyne/cm^2)	$Y/5$ (10^{11}dyne/cm^2)	Theoretical strength (10^{11}dyne/cm^2)	Practical strength (10^{11}dyne/cm^2)
Iron	16	3	3	.085
Copper	19	4	3	.049
Silicon	18	4	3	.062
Glass	7	1	4	.002

The Origin of Fracture Mechanics

Large advances in the understanding of fracture have tended to follow great public disasters. In 1919, a molasses tank 50 feet high and 90 feet wide burst in Boston, killing twelve people and several horses. The court auditor concluded that "the only rock to which he could safely cling was the obvious fact that at least one-half of the scientists must be wrong."[18] The most important case in this century occurred during World War II. Wartime demands for ocean-going freighters led to the production of the Liberty ship, the first ship to have an all-welded hull. Of the nearly 4700 Liberty ships launched during the war, over 200 suffered catastrophic failure, some splitting in two while lying at anchor in port, and over 1200 suffered some sort of severe damage due to fractures. The discipline of fracture mechanics emerged from these catastrophes. The all-welded ships were redesigned, eliminating, for example, sharp corners on hatches, and systematic procedures were developed for testing the fracture resistance of materials. In the early 1950s, failure by fracture cursed the airline industry's efforts to establish passenger service using jet aircraft. Ill-placed rivet holes destroyed two of Britain's Comet aircraft, and played a role in moving the center of civilian jet aircraft production to the United States. Aircraft are now subject to a systematic program of inspection that acknowledges that every structure has flaws, but that flaws greater than a certain size are intolerable. Procedures have continued to evolve in response to accidents, most recently after an incident where part of the top of the fuselage of an airliner peeled off during flight.

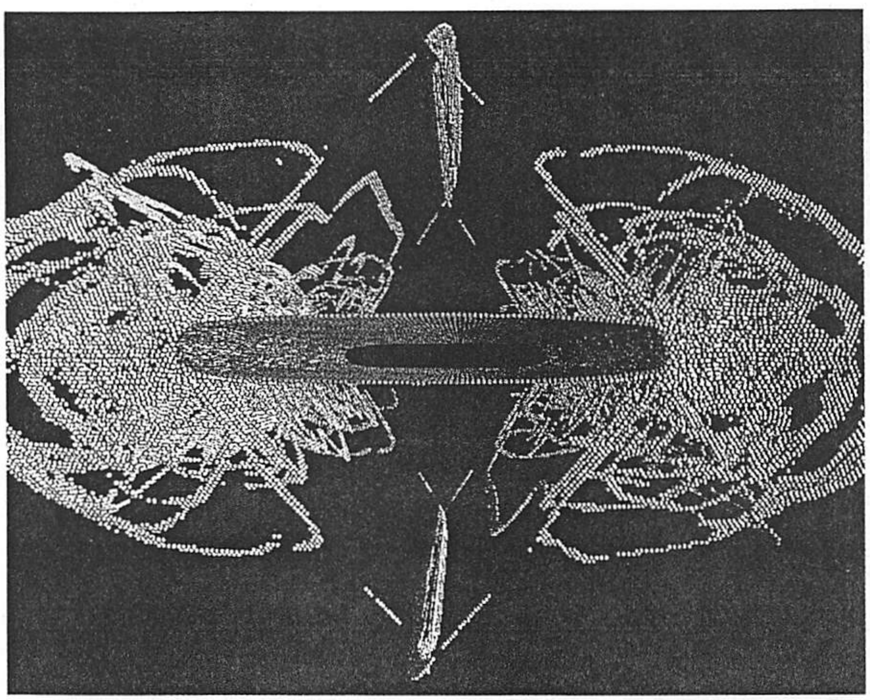


and Robb Thomson showed[4] how to estimate whether the crack will move forward in response to such a stress, or whether instead a crystal dislocation will pop out of the crack tip, causing the tip to become blunt. Figure 2 shows a very large computer simulation in which an elliptical crack is placed in copper, one of the most pliable of metals. The tip of the crack spawns clouds of dislocations, appearing as stringy white vortex cores, which travel off into the crystal in unexpected directions and provide strong impediments to further motion.

Brittleness and ductility, in fact, are not inherent in the atoms that make up a solid. Most solids have a definite temperature at which they make a transition from brittle to ductile behavior. For silicon, this temperature is around 500 °C.[3] This transition is not as well understood as the more familiar equilibrium phase changes.

Crack Dynamics

Cracks would cause no one any trouble if they never moved, so it is natural to investigate their dynamics in some detail. The first calculations along these lines were carried out by Neville Mott in response to the Liberty ship



SIMULATION OF A DUCTILE MATERIAL with 35 million atoms. An elliptical crack (outlined in red atoms) in a 0.1 μm thick copper sheet is placed under tension in the vertical direction. As the crack attempts to propagate horizontally, it emits clouds of dislocations (white), some of which have collided directly above the crack. Only the atoms at the surface of the crack, or within the cores of dislocations, are depicted. The supercomputer calculation was performed by Shujia Zhou, David Beazley, Peter Lomdahl, and Brad Holian of the Theoretical Division at Los Alamos National Laboratory. FIGURE 2 disasters during World War II (see the box on page 26); Mott's work led to an amazingly successful scaling theory described in the box at right.

The scaling theory stood up remarkably well to increasingly sophisticated mathematical improvement. Its only defect was that it never agreed with experiment.[5] All equations of motion for cracks predicted that cracks should accelerate up to the Rayleigh wave speed—the speed of sound traveling over a flat surface, or of earthquakes traveling over the surface of Earth. Experiments dating back as far as 1937[6] showed that cracks in glass went, at most, at half this speed. For a field in which the main goal is to keep large tankers from splitting in half, the question of precisely how fast a crack runs across the hull seems rather esoteric. But if the goal is a detailed understanding of the conditions under which a crack can move, getting the velocity right is a necessary first step.

One hint that the motion of cracks might be more complicated than that of particles moving in straight lines came from examining the new surfaces that cracks left behind them. The surfaces often have visibly rough features (as shown in figure 3), which develop only after the crack has traveled some distance. Several years ago, with Harry Swinney and Steve Gross, we developed a technique that made it possible to measure the velocity of a crack twenty million times per second, tens of thousands of times in succession, and to an accuracy of around twenty meters per second.[7] The method involved depositing a very thin layer of aluminum on a Plexiglas or glass sample, and then monitoring its resistance as a crack ran through it. The great detail in data from our experiments on long samples of brittle materials, prepared with a notch sawed in one side, clearly showed that crack motion in such materials could pass through three distinct phases:

▷ Birth: Long, sharp initial notches turn into rapidly running cracks at low stresses, while short blunt notches refuse to move until the stress energy density is as much as ten times greater. However, in almost all cases, cracks accelerate in less than a microsecond to a substantial fraction of the speed of sound, at least 200 meters per second.

▷ Childhood: The early phases of crack motion involve calm and efficient progress through the sample. The new surfaces left behind the crack are smooth and mirrorlike, as shown on the lower right-hand part of figure 4; the crack velocity is smoothly and slowly increasing, as shown in the left half of figure 4. For long, sharp initial cracks, the entire sample is severed in this fashion.

▷ Crisis: However, cracks that pass beyond a critical threshold in velocity begin to buck and plunge, as shown by the black curve in the left half of figure 4. They leave increasingly rough surfaces in their wake, shown in figure 3, and their velocities undulate at frequencies of hundreds of kilohertz.

Thus, cracks in brittle materials suffer a dynamical instability, which makes them unable to accelerate up to the high velocities predicted by classic theories of dynamical fracture.

Origin of dynamical instability

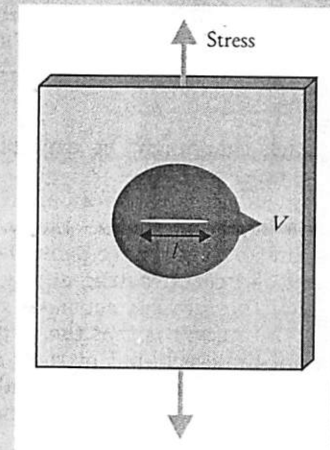
Lurking behind the theories of dynamical fracture have always been certain puzzling contradictions. Elizabeth Yoffe carried out the first detailed calculation of dynamical fracture,[8] and pointed out that cracks are strongly influenced by special relativity—as they approach not the speed of light, but that of sound. Stresses in the neighborhood of the crack adopt a universal form near the tip, and this universal singularity contracts in the direction of rapid motion. Yoffe observed that at around 60% of the speed of sound, lobes developed in the stress field surrounding

How Cracks Grow

A crack of length l grows at rate v in a plate. (See the figure below right.) There are three important energies:

▷ Potential energy: The potential energy decreases as the crack extends, and since the size of the region where this happens scales as l^2, the potential energy released scales as $-l^2$; it also scales as the square of the applied stress.

▷ Fracture energy: Making the crack move forward requires breaking bonds, creating new surfaces and generating heat; the energy required scales as the length of the crack, l.

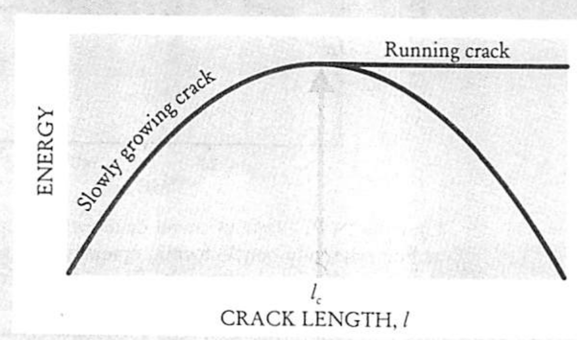


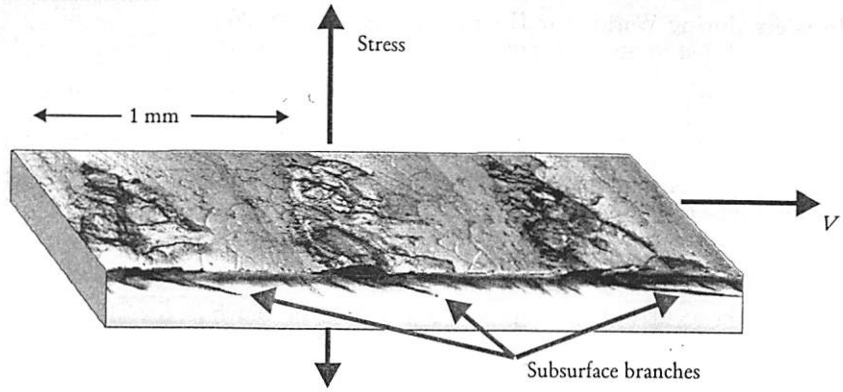
▷ Kinetic Energy: The total kinetic energy due to the motion of the crack scales as l^2v^2, since the amount of mass that moves as the crack opens scales as l^2.

For very slowly moving cracks, only potential and fracture energies are important, and the sum of these energies as a function of l is shown in the sketch at the bottom of this box. Since potential energy decreases as l^2 and fracture energy increases as l, for very small cracks the fracture energy is always larger, and the total energy increases with l. This is a fortunate fact, or else all solid objects would be completely unstable if subjected to the slightest mechanical stress. But eventually the potential energy overwhelms the fracture energy at the critical crack length, l_c, called the Griffith point, and from here on, more energy is released than consumed by crack extension. Now extension is rapid and spontaneous. Since the sum of fracture and potential decreases as $(l - l_c)^2$ for $l > l_c$ and energy is conserved by converting potential to kinetic energy, one easily finds that the velocity of the crack must be

$$v(t) = v_{max}(1 - l_c/l) \qquad (3)$$

The critical stress needed to snap a body with a crack of size l scales as $l^{1/2}$. Like the results of many other scaling arguments, equation 3 is better than one has any right to expect. Fifteen years of careful mathematical work, documented in the book by L. Ben Freund,[8] extracts the same formula from a remarkably general boundary-value problem of classical elasticity. In the rigorous formulation v_{max} turns out to be the Rayleigh wave speed.





WHEN CRACKS EXCEED a critical velocity in Plexiglas, the fracture surface acquires visible roughness with a wavelength of approximately one millimeter. The roughness results from the violent creation of subsurface branches. The amplitude of the surface roughness is two orders of magnitude smaller than the depth of the subsurface branches. FIGURE 3

the crack that might be expected to force it to deviate from a straight line.

Moving cracks are even more prone to instability than Yoffe's calculation shows. Emily Ching, Hiizu Nakanishi and James Langer[9] have pointed out that, if one looks out in front of a crack moving at any speed and asks in what direction the stresses act most strongly to tear material apart, the answer is that the largest stresses are straight ahead of the crack, but at right angles to its direction of motion. According to this calculation, cracks should always move perpendicular to themselves, and stable motion should be impossible.

Thus, from the viewpoint of classical elasticity, assuming that cracks are stable leads to an equation of motion the cracks do not obey, and probing stability of cracks more deeply makes it seem puzzling that they are able to propagate at all.

These difficulties have partly been answered by calculations at the atomic scale. There is a very special set of forces between atoms, discovered by Leonid Slepyan,[10] which makes it possible to find analytical solutions for cracks moving in lattices. The behavior of cracks in these models has several surprising features, but all of them are mirrored in the experiments. These features are:[11]

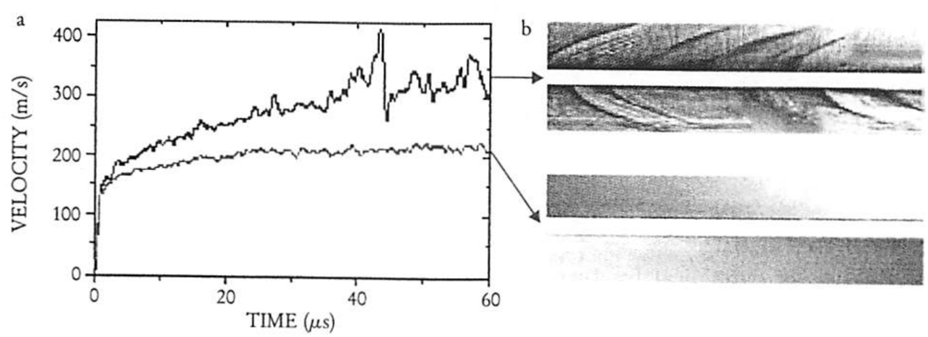
▷ Birth: There is a range of velocities at which steady crack motion is forbidden. The range starts at zero and lasts until around 20% of the speed of sound, after which crack motion becomes possible.

▷ Childhood: Following the forbidden band, a range of velocities exists for which steady crack motion is allowed and perfectly stable. At exactly the same externally applied stress, however, a stationary crack could also be stable.

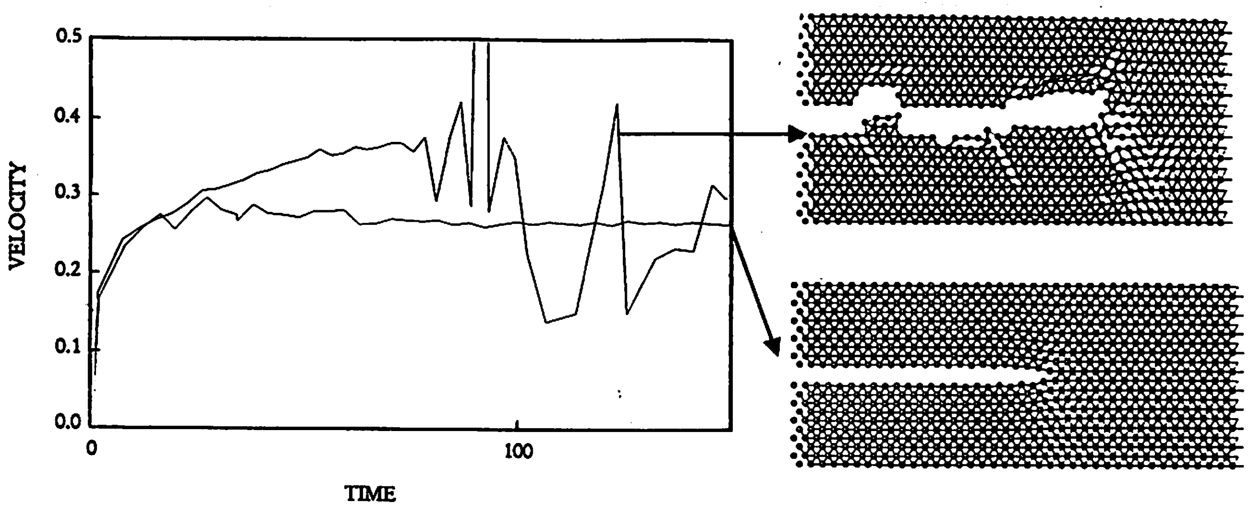
▷ Crisis: Above a critical velocity, steady crack motion becomes unstable.

Careful investigation of solutions of these models shows both how to defeat the instabilities lurking behind continuum theory and how the crack tip disintegrates when pressed too hard. For a range of low velocities, steady, moving crack solutions are completely stable. As the crack speeds up, the relativistic contraction discovered by Yoffe becomes more and more important, until eventually horizontal bonds above the crack line begin to snap. Whether the crack arrives at this point depends, of course, on how hard it is being pulled; once it happens, however, perfect steady motion along a line becomes impossible. Simulations, such as that in the upper right of figure 5, have shown that the crack might decide to build treelike patterns of subsurface cracks once steady motion becomes impossible.

Having seen fracture trees in simulation,[12] we set out to find them in experiment. Our first try involved an ill-considered attempt to sand down a piece of Plexiglas that nearly set a milling machine on fire (Fineberg takes no responsibility for Marder's fine efforts in the laboratory), but soon we did better,[13] as shown in the upper right of figure 4. So extensive does the network of branches in Plexiglas become that it explains the inability of cracks to accelerate to the predicted limiting speed.[14] Once instability sets in, pulling more on a crack simply makes it dig in its heels harder, generating that much more subsurface damage but scarcely leading to any more acceleration. In some simulations, as shown on the left side of figure 5, pulling harder on a crack can actually



CRACKS IN PLEXIGLAS travel differently, depending on the force with which they are pulled. **a:** For relatively gentle forces, cracks travel calmly, their velocity increasing smoothly and slowly with time (red trace). Beyond a critical velocity, cracks move with wildly undulating speed (black trace). **b:** Slowly moving cracks tend to leave smooth surfaces (lower image). Cracks propagating at speeds above the critical velocity leave a thicket of small branches penetrating the surface behind them (upper image). FIGURE 4



COMPUTER SIMULATIONS in a simple atomic-scale model display a transition between smoothly moving cracks (red trace and lower right) and a violent, branching instability (black trace and upper right) that is surprisingly similar to experiment. Just as in experiments, the transition is a function of the energy stored per unit length to the right of the crack. The velocity, v, is measured relative to the shear wave speed and the time, t, relative to the vibrational period of the atomic bonds. FIGURE 5

slow it down. Over 90% of the energy being fed to the tip of a crack can be consumed by subsurface instability.

The key and the glass

Engineering fracture mechanics has had enormous success in improving the safety of structures in this century. Attempts to understand the mechanism of fracture at an atomic level have not yet had a comparable impact. The main reason is not hard to find.

Structural materials in common use have evolved from a process of trial and error that has occupied thousands of years.[2] At a microscopic level, they are incredibly complex. For example, Plexiglas, which in figures 4 and 5 we blithely compare with a triangular lattice, is actually composed of molecules a million units long, tangled about one another in an amorphous web. Iron only becomes useful after the addition of subtle impurities in elaborate industrial processes. The most widely used structural material of all—wood—obtains marvelous mechanical properties in ways that humans have not yet learned to imitate.

Green twigs bend and dry twigs snap, but while the dislocations shown in figure 2 provide an explanation for the ductility of copper crystals, they help little with something as noncrystalline as Plexiglas, let alone a tree. Almost all of solid-state physics rests upon calculations carried out in crystals, but whereas the perfect crystal makes a wonderful electrical conductor, it makes a lousy brick. The largest remaining challenge for physics in the study of how things break is to begin to bridge the gap between simple model systems and the rich diversity of the real world. Computer simulations have an important role to play[15,16] and can treat an imposing number of atoms, but conceptual understanding of how to reason from the small to the large will play an equally important role. The computer can treat 100 million atoms for a few times 10^{-12} seconds, but we need to understand 10^{23} atoms on time scales of minutes or years.

Eugene Wigner remarked that solid-state physics "deals in a scientific way with those subjects with which we must deal in our everyday experience. For example, we are never afraid when dropping a key that it will fly to pieces, as a glass would."[17] This first fact that children learn about solids seems, however, to be one of the last that scientists will be able to explain. A microscopic picture of the strength of solids has begun to emerge, but much more remains to be learned.

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