For Michael William Ov

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ılves the further necessity of adequate vehicle or ground-based.

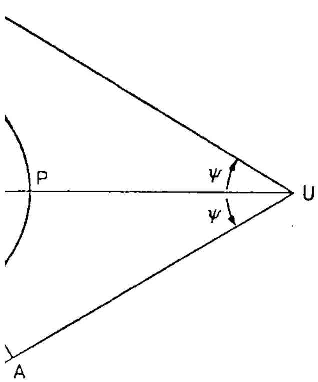
fore is the focusing effect of the target hit the target body, it is not necessary should intersect the planet but only ic encounter orbit should touch the ıg as the asymptote of the hyperbolic e OA from the centre of the planet, lius of the planet, the 'collision radius'

$$) \sin\left[\pi - \cos^{-1}\left(\frac{1}{e}\right)\right]$$

the hyperbolic excess; hence

$$\frac{1}{\ }. \qquad (11.95)$$

ıs be much larger than the true radius the giant planets Jupiter and Saturn.



11.14

11.4.5 The fly-past as a velocity amplifier

In recent years a planetary fly-past has been used to alter the trajectory of a vehicle so that its modified heliocentric orbit takes it to some other planet. For example the fly-past of Venus by Mariner 10 took it inwards to make three subsequent fly-pasts of Mercury; the Voyager fly-pasts of Jupiter took them way out to Saturn and beyond. In this section we look at the way in which a close encounter with a planet may be used to change a space probe's heliocentric velocity, using a number of results obtained in previous sections.

Consider the simple case of a spacecraft travelling in a Hohmann cotangential ellipse between the orbits of Earth and Jupiter. The hyperbolic excess velocity V with which the spacecraft enters the sphere of influence of Jupiter is then given approximately by equation (11.22),

$G = 6.7\times10^{-11}$
$M = 1.99\times10^{31}$

$$V = \left(\frac{\mu}{a_2}\right)^{1/2}\left[1-\left(\frac{2}{1+a_2/a_1}\right)^{1/2}\right] \qquad (11.96)$$

where $\mu = GM$ (G being the constant of gravitation and M the Sun's mass) and a_1 and a_2 are the orbital radii of Earth and Jupiter respectively. We are assuming that Jupiter overtakes the spacecraft, which at this time is travelling almost tangential to Jupiter's orbit.

Now the Jovian sphere of influence radius r is given by

$$r = \left(\frac{m}{M}\right)^{2/5} a_2 \qquad (11.97)$$

where m is the mass of Jupiter. Putting in the relevant values we find that $r = 0{\cdot}322$ AU. The radius of Jupiter in these units is $r_J = 0{\cdot}000477$. By equation (4.91) we can then obtain a value for a, that is, from

$$V^2 = \mu_J\left(\frac{2}{r}+\frac{1}{a}\right) \qquad (11.98)$$

where $\mu_J = Gm$, by putting in the relevant values from (11.96) and (11.97).

The vehicle now performs a hyperbolic fly-past of Jupiter within its sphere of influence. Its entry into the sphere of influence may be chosen so that its perijove distance r_p is not much more than the radius of the planet r_J.

By using the relation $r_p = a(e-1)$, we obtain

$$e = \frac{r_p}{a} + 1.$$

We also have

$$b = a(e^2-1)^{1/2}. \qquad (11.99)$$

Now the asymptotes of the hyperbolic encounter are given by

$$\tan\psi = \pm b/a \qquad (11.100)$$

and it is readily seen from figure 11.15 that the effect of the encounter is to

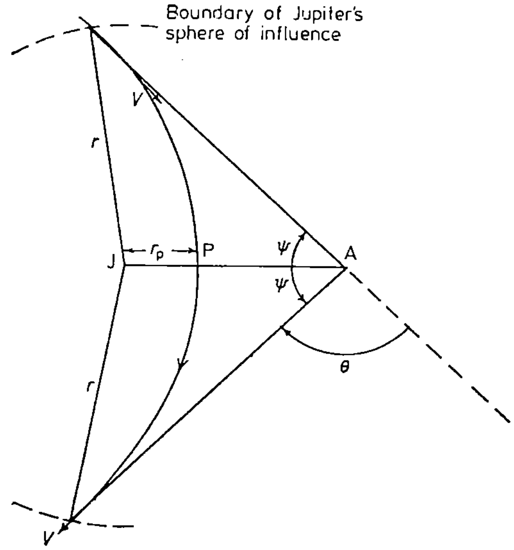


Figure 11.15

rotate the direction in which the vehicle travels through an angle θ given by

$$\theta = \pi - 2\psi. \qquad (11.101)$$

Then by equations (11.99), (11.100) and (11.101), we may write

$$\tan\left(\frac{\theta}{2}\right) = (e^2 - 1)^{-1/2}. \qquad (11.102)$$

Substituting numerical values for a_1, a_2, Gm, GM, r and r_J, it is found approximately that

$$\left.\begin{aligned} V &= 1{\cdot}19 \text{ AU/year} \\ \theta &= 160{\cdot}3^\circ \end{aligned}\right\} \qquad (11.103)$$

Circular velocity at Jupiter's distance from the Sun is $V_c = 2{\cdot}76$ AU/year. The velocity of escape from the Solar System (at Jupiter's distance) is therefore $V_c\sqrt{2}$ (i.e. 3·90 AU/year). We see then from equation (11·103) that the effect of the encounter is to eject the spacecraft from Jupiter's sphere of influence in almost the opposite direction to which it entered and with a velocity which, added to Jupiter's orbital velocity, gives a speed greater than the velocity of escape from the Solar System.

The effect could have been further amplified by firing the vehicle's engine at perijove to increase the hyperbolic excess velocity in accordance with the principles of section 11.4.1. It is therefore seen that using a planetary mass as a velocity amplifier has practical applications.

Problems

The data in the appendices should be u

11.1 What effect is produced in the v
free space by (i) doubling the exhaust

11.2 A rocket with an initial mass of
of the rocket is 2000 m s^{-1} and the f
atmospheric drag, calculate the burn-o
when it was fired vertically upwards un
a constant = 981 cm s^{-2}).

11.3 It is proposed to put the upper
orbit in which the velocity is 7·73 km s
velocity of 3000 m s^{-1} (twice that of th
mass ratio R, and the ratio of the fu
fuelled lower-stage mass is 0·15, calcul
the empty upper stage that goes into o
drag losses).

11.4 Compare the velocity increment s
heliocentric circular orbit to one of 40
(ii) by using a cotangential bi-elliptic tr

11.5 Compare the transfer times in p

11.6 Two circular coplanar heliocen
moving in the inner orbit uses its mot
times the velocity increment required t
tangential elliptic transfer orbit as far o
to the outer distance is achieved?

11.7 In the preceeding problem wha
the cotangential elliptic transfer orbit, (
orbit with the outer circular orbit, to pla

11.8 Two circular heliocentric orbits h
of 5°. It is proposed to transfer a vehi
path into the inner one by applying two
Should the change in orbit inclination b
in fuel is to be made? Calculate the sa
decision is made.

11.9 Suppose that in the Moon-shot
of arc. Find to the first order the resultin
of the semimajor axis, the apogee distan

11.10 Two asteroids move in circula
elements:

Asteroid	Orbital radius (AU)	
A	2	1
B	3·5	2

An absent-minded asteroid prospector v
greatest economy in fuel, to B. Find his
B he discovers that he has left his Geig
is his minimum waiting time on B if the r
condition? (Neglect the asteroids' gravit

11.11 An interplanetary probe leave
6630 km with a tangential velocity of 12 k