; an indication of upper mantle

ɛ waves on a spherical earth, **1,**

G., 1969. Fine structure of the

;mic waves near 110°: is structure
. *J. R. astr. Soc.*, **16,** 475.
ted effects, Ch. 3, pp. 47–74 in
Prentice-Hall, Inc., Englewood

pa process for analysis of seismic

ınalysis of PKP phases and their

ected from discontinuities within

;urface ' of the earth: P′ P′ P′ P′,

l. Discontinuities in the earth's
energy, *Bull. seism. Soc. Am.*, **51,**

: Gutowski, P. R., 1973. Seismic
ıe Hawaiian linear volcanic chain,

n of seismic velocities—a review,

Dynamic correlation analysis,

ffect of temperature and partial
le binary system, *J. geophys. Res.*,

›ectra-digital computer derivation
ısics, **34,** 859.
ıic waves from 0 to 150 km depth
and the Atlantic-Indian rise, *Am.*

alternative to P′ & P′ reflections,

lection of P′ P′ seismic waves from
., **75,** 5713.

Geophys. J. R. astr. Soc. (1974) **36,** 33–42.

Membrane Tectonics

D. L. Turcotte*

(Received 1973 June 21)

Summary

Due to the ellipticity of the Earth the lithosphere must deform when its latitude changes. In this paper we approximate the lithosphere with an equivalent spherical dome and determine the membrane stresses associated with this deformation. Stresses of the order of kilobars are found. Stresses of this magnitude may result in propagating fractures in the lithosphere. Ocean island chains and graben valleys may be the result of such propagating fractures.

Introduction

The basic hypothesis of plate tectonics is that the outer shell of the Earth (the lithosphere) is broken up into a number of rigid plates which are in relative motion with respect to each other. If the Earth was a perfect sphere these surface plates would be free to slide about without deformation. However, to a good approximation, the Earth is an oblate spheroid with an ellipticity $\varepsilon = 0{\cdot}00335$. Due to the equatorial bulge any change in latitude requires a deformation of the surface plate. A plate moving away from the equator will have its principal radii of curvature increased. A plate moving towards the equator will have its principal radii of curvature decreased. It has been shown by Turcotte & Oxburgh (1973) that the magnitude of the stresses associated with a change in latitude may be sufficient to fracture the surface plates.

Since the rigid surface plates are thin (50–75 km thick) compared with the radius of the Earth they will behave like thin shells when deformed. Bending stresses can be neglected compared with the membrane stresses. If the radii of curvature of an unstressed thin shell are increased the edge will be in tension and the interior in compression. If the radii of curvature are decreased the edge will be in compression and the interior will be in tension.

The surface plates move with velocities in the range 1–10 cm/year. Therefore, a significant change of latitude requires about 10^8 yr. An important question is whether the surface plates will behave as an elastic medium on this long time scale. If plastic yielding occurs, the membrane stresses will be relieved. Certainly, at modest depths in the plates the temperatures will be sufficiently high for plastic yielding to be expected. However, to relieve the membrane stresses plastic flow must occur across the entire thickness of the plates. There is no evidence that the cold, brittle near surface rocks yield plastically on geological time scales. The presence of major fault systems shows that brittle fractures occur when displacements of the order of centimetres per year occur.

There is also observational evidence that the interior of the surface plates fracture under tension. Structural and petrological studies of the Hawaiian Islands (Jackson & Wright 1970; Green 1971) indicate that they are a result of magma flows through a propagating tensional fracture. Graben structures such as the Rhine Valley are

* Present address: Department of Geological Sciences, Kimball Hall, Cornell University, Ithaca, New York 14850.

also, apparently, the result of tensional fractures of the surface plate with finite extension (Illies 1970).

It is the purpose of this paper to determine the magnitude and distribution of membrane stresses due to the change in latitude of a surface plate. The associated gravity anomalies will also be determined.

The spheroid

The accepted form of the spheroid (Wilkins 1965) has an equatorial radius $a = 6378{\cdot}160$ km and an ellipticity $\varepsilon = 0{\cdot}0033529$. The two principal radii of curvature at any point on the spheroid are given by (Bomford 1952)

$$\rho = \frac{a(1-\varepsilon)^2}{(\cos^2\gamma + [1-\varepsilon]^2 \sin^2\gamma)^{\frac{3}{2}}} \tag{1}$$

$$\nu = \frac{a}{(\cos^2\gamma + [1-\varepsilon]^2 \sin^2\gamma)^{\frac{1}{2}}} \tag{2}$$

where γ is the latitude of the point. The radius of curvature ρ is the radius of the circle tangent to the spheroid along a meridian of longitude. The radius of curvature ν is the radius of the circle tangent to the spheroid along a parallel of latitude. At the equator ($\gamma = 0$) the radii of curvature are $\rho = a(1-\varepsilon)^2 = 6335{\cdot}46$ km and $\nu = a$. At the poles ($\gamma = \pi/2$) the radii of curvature are equal and

$$\rho = \nu = a/(1-\varepsilon) = 6399{\cdot}62 \text{ km}.$$

The radii of curvature of the spheroid as a function of latitude are given in Fig. 1.

The model

Each element of a surface plate is formed with radii of curvature corresponding to the radii of curvature of the point on the spheroid where that element of the plate

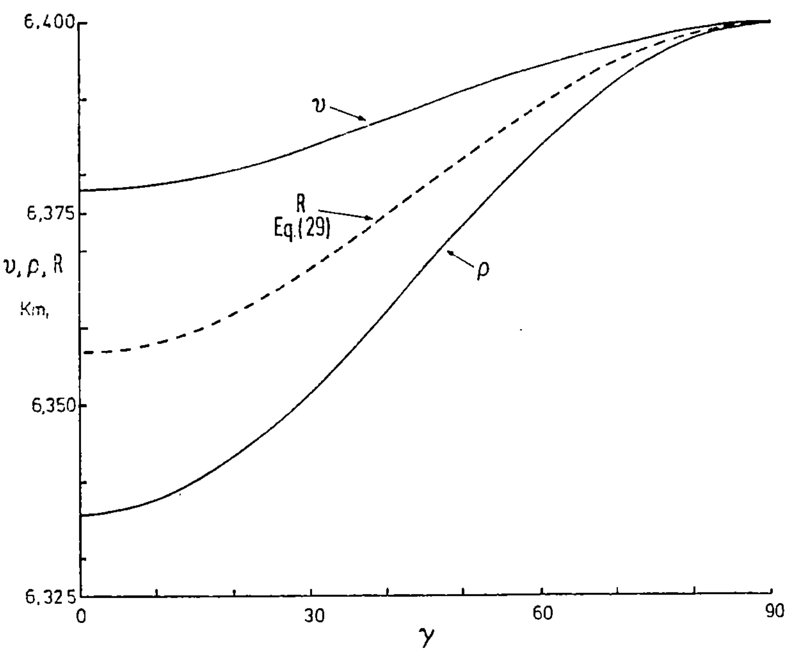

FIG. 1. The principal radii of curvature of the Earth as a function of latitude.

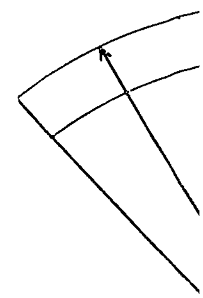

FIG. 2. Cross-section of tł
positions. The radius of th

is formed. When the latitude
element must change to corres|
spheroid. The resultant defor
plate.

Since the surface plates hav
are formed at different latitud
a very complex problem. For
We will consider a circula
Initially the spherical dome ha
shell is deformed into a spheri
curvature $R+\Delta R$. The defor
(pressure) on the shell. Both t
taken to be positive outwards a
The deformation of the shell
of the undeformed spherical sh
the applied surface force and th
of the angle ϕ. After deforma
the centre of the deformed shell
of the deformed shell is given

$[(R+w)\sin\phi$

as illustrated in Fig. 2. Assum

We will show that the membra

Formulation of the problem

Since the thickness of the s
deformation of the shell can
1959). Bending stresses can be
(membrane stresses) in the sh
meridians is σ_ϕ and the stress

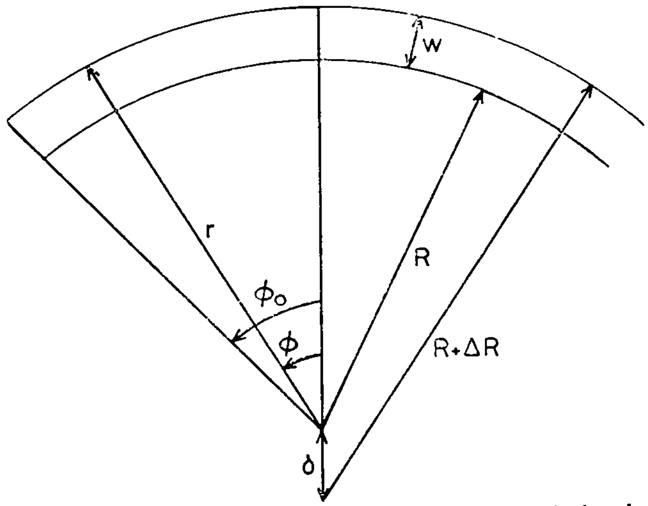

FIG. 2. Cross-section of the spherical dome showing the original and deformed positions. The radius of the dome is increased R to $R+\Delta R$, the centre is displaced downwards a distance δ.

is formed. When the latitude of the element changes, the radii of curvature of the element must change to correspond to the radii of curvature at the new point on the spheroid. The resultant deformation of the plate leads to membrane stresses in the plate.

Since the surface plates have irregular shapes and different elements of each plate are formed at different latitudes, a determination of the membrane stresses can be a very complex problem. For this reason we introduce a simplified model.

We will consider a circular segment of a spherical shell, i.e. a spherical dome. Initially the spherical dome has a radius R and is unstressed. We assume that the shell is deformed into a spherical dome with a slightly larger (or smaller) radius of curvature $R+\Delta R$. The deformation takes place because of a radial surface force (pressure) on the shell. Both the surface force, p_r, and the radial deformation, w, are taken to be positive outwards and negative inwards.

The deformation of the shell is based on a co-ordinate system centred at the centre of the undeformed spherical shell as shown in Fig. 2. Because of the symmetry both the applied surface force and the deformations as well as the stresses are only functions of the angle ϕ. After deformation to a spherical dome with a slightly larger radius the centre of the deformed shell is assumed to be displaced a distance δ. The position of the deformed shell is given by

$$[(R+w)\sin\phi]^2+[(R+w)\cos\phi+\delta]^2 = (R+\Delta R)^2 \tag{3}$$

as illustrated in Fig. 2. Assuming that $\Delta R/R \ll 1$ and $\delta/R \ll 1$ we find that

$$w = \Delta R-\delta\cos\phi. \tag{4}$$

We will show that the membrane stresses are a function of ΔR but not of δ.

Formulation of the problem

Since the thickness of the spherical dome is small compared with its radius the deformation of the shell can be determined using membrane theory (Novozhilov 1959). Bending stresses can be neglected compared with the tension and compression (membrane stresses) in the shell. The stress in the shell in the direction of the meridians is σ_ϕ and the stress normal to the meridians is σ_ϕ (positive stresses are

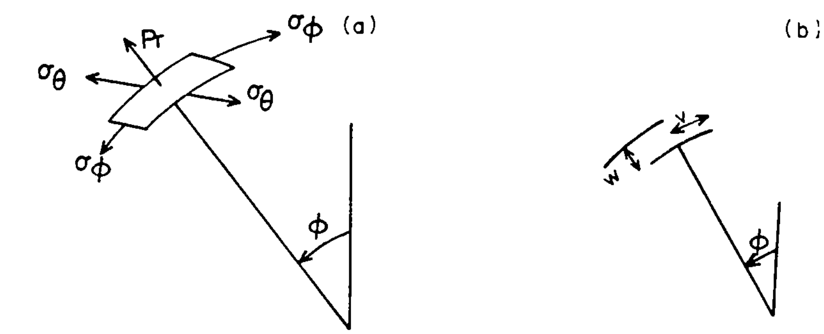

FIG. 3. (a) Force balance on a segment of the spherical dome. (b) Deformation of a segment of the spherical dome.

tensions, negative compressions). Because of the symmetry of the loading and deformation there are no shear stresses in these directions. The equilibrium conditions on an element of the shell require that

$$\sigma_\phi + \sigma_\theta = \frac{R}{h} p_r \tag{5}$$

$$\frac{d}{d\phi}(\sin\phi\,\sigma_\phi) = \cos\phi\,\sigma_\theta \tag{6}$$

where h is the thickness of the shell. The force balance is illustrated in Fig. 3(a).

Using Hooke's law the displacements in the radial direction w (positive outward) and in the meridianal direction v (positive for increasing ϕ) are related to the stresses by

$$v\cot\phi + w = \frac{R}{E}(\sigma_\theta - \nu\sigma_\phi) \tag{7}$$

$$\frac{dv}{d\phi} + w = \frac{R}{E}(\sigma_\phi - \nu\sigma_\theta) \tag{8}$$

where E is Young's modulus and ν is Poisson's ratio. The displacements are illustrated in Fig. 3(b).

We assume that there is no load on the edge of the shell. This requires that the boundary condition

$$\sigma_\phi = 0 \quad \text{at} \quad \phi = \phi_0 \tag{9}$$

be satisfied.

Combining equation (5)–(8) gives a single equation for the surface force in terms of the radial displacement, the result is

$$\frac{d^2 p_r}{d\phi^2} + \cot\phi\frac{dp_r}{d\phi} + (1-\nu)p_r = \frac{Eh}{R^2}\left(\frac{d^2 w}{d\phi^2} + \cot\phi\frac{dw}{d\phi} + 2w\right). \tag{10}$$

Substitution of the radial displacement from equation (4) gives

$$\frac{d^2 p_r}{d\phi^2} + \cot\phi\frac{dp_r}{d\phi} + (1-\nu)p_r = 2\frac{Eh\Delta R}{R^2}. \tag{11}$$

The solution of this equation gives the distribution of surface force required to deform a spherical dome of radius R into a spherical dome with a slightly larger (smaller) radius $R+dR$. Introducing

$$p_r = p_{rh} + \frac{2Eh\Delta R}{R^2(1-\nu)}, \qquad \xi = \cos\phi, \qquad \nu = 1-\alpha(1+\alpha) \tag{12}$$

reduces equation (11) to a homogeneous equation

$$(\xi-1)\frac{d^2 p_{rh}}{d\xi^2} + 2\xi\frac{dp_{rh}}{d\xi} - \alpha(\alpha+1)p_{rh} = 0 \tag{13}$$

which is Legendre's equation. The solution for the surface force is therefore given by

$$p_r = \frac{2Eh\Delta R}{R^2(1-\nu)} + AP_\alpha(\xi) \tag{14}$$

where $P_\alpha(\xi)$ is the Legendre function of the first kind of order α. The second solution $Q_\alpha(\xi)$, the Legendre function of the second kind of order α, must be discarded since it is singular for $\xi = 1$, $\phi = 0$. The constant A is to be evaluated from the boundary condition, equation (9). With a specific exception which we will consider, the Legendre functions of non-integer order are not readily evaluated.

Shallow shell approximation

If ϕ_0 is small a solution of equation (11) can be obtained by expanding in powers of ϕ. This is equivalent to the shallow shell approximation in thin shell theory (Reissner 1946). The solution of equation (11) which satisfies the boundary condition equation (9) gives

$$p_r = \frac{hE\Delta R}{2R^2}(\phi^2 - \tfrac{1}{2}\phi_0{}^2) \tag{15}$$

$$\sigma_\phi = \frac{E\Delta R}{8R}(\phi^2 - \phi_0{}^2) \tag{16}$$

$$\sigma_\theta = \frac{E\Delta R}{8R}(3\phi^2 - \phi_0{}^2). \tag{17}$$

These results are correct to order $\phi_0{}^2$.

For positive ΔR the radial surface force is inward for $\phi < \phi_0/\sqrt{2}$, the radial surface force is outward for $\phi_0 > \phi > \phi_0/\sqrt{2}$. The interior of the shell, $\phi < \phi/\sqrt{3}$, is in compression. For $\phi_0 > \phi > \phi_0/\sqrt{3}$ the tangential stress σ_θ is a tension. The maximum stress is the tension at the edge of the shell which is given by

$$(\sigma_\theta)_{\phi=\phi_0} = \frac{E\Delta R}{4R}\phi_0{}^2. \tag{18}$$

In the shallow shell approximation the stresses and surface force are independent of Poisson's ratio. Before applying these results to the Earth's lithosphere we will determine the applicability of the approximation by comparing the results with a solution valid for arbitrary ϕ_0.

Solution for Poisson's ratio equal to 1/4

For $\nu = \frac{1}{4}$ we find from equation (12) that $\alpha = \frac{1}{2}$. In this case an analytic solution of the problem can be obtained for any value of ϕ_0. Taking $\nu = \frac{1}{4}$ is a good approximation for the Earth's lithosphere since seismic studies (Bullen & Haddon 1967) show that $\nu = 0{\cdot}217$ in the upper mantle below the Moho. The Legendre function of the first kind for order $\frac{1}{2}$ is given by

$$P_{\frac{1}{2}}(\cos\phi) = 2E\left(\left[\frac{1-\cos\phi}{2}\right]^{\frac{1}{2}}\right) - K\left(\left[\frac{1-\cos\phi}{2}\right]^{\frac{1}{2}}\right) \tag{19}$$

where $E(k)$ and $K(k)$ are the complete elliptic integrals of the first and second kind

$$E(k) = \int_0^{\pi/2} (1-k^2\sin^2\alpha)^{\frac{1}{2}}\,d\alpha \tag{20}$$

$$K(k) = \int_0^{\pi/2} (1-k^2\sin^2\alpha)^{-\frac{1}{2}}\,d\alpha\,. \tag{21}$$

Using the derivatives

$$\frac{dE}{dk} = \frac{1}{k}(E-K), \quad \frac{dK}{dk} = \frac{1}{k}\left(\frac{1}{1-k_2}E-K\right) \tag{22}$$

the boundary condition, equation (9), can be satisfied with the result

$$p_r = \frac{hE\Delta R}{3R^2}\left[8 - \frac{5\{2E(k)-K(k)\}}{(2+\cot\phi_0)E(k_0) - \dfrac{(2-\cos\phi_0)}{2(1-\cos\phi_0)}K(k_0)}\right] \tag{23}$$

$$\sigma_\phi = \frac{4E\Delta R}{3R}\left[1 - \frac{(2+\cot\phi)E(k) - \dfrac{(2-\cos\phi)}{2(1-\cos\phi)}K(k)}{(2+\cot\phi_0)E(k_0) - \dfrac{(2-\cos\phi_0)}{2(1-\cos\phi_0)}K(k_0)}\right] \tag{24}$$

$$\sigma_\theta = \frac{E\Delta R}{3R}\left[4 - \frac{2(1-2\cot^2\phi)E(k) - \dfrac{(1-3\cos\phi)}{(1-\cos\phi)}K(k)}{(2+\cot^2\phi_0)E(k_0) - \dfrac{(2-\cos\phi_0)}{2(1-\cos\phi_0)}K(k_0)}\right] \tag{25}$$

where

$$k = \left(\frac{1-\cos\phi}{2}\right)^{\frac{1}{2}}, \quad k_0 = \left(\frac{1-\cos\phi_0}{2}\right)^{\frac{1}{2}}.$$

Values of the complete elliptic integrals are tabulated by Abramowitz & Segun (1965).

Again the maximum stress is the tension at the edge of the shell and this is given by

$$(\sigma_\phi)_{\phi=\phi_0} = \frac{2E\Delta R}{3R}\left[\frac{(3+4\cot^2\phi_0)E(k_0) - \dfrac{(3+\cos\phi_0)}{2(1-\cos\phi_0)}K(k_0)}{(2+\cot^2\phi_0)E(k_0) - \dfrac{(2-\cos\phi_0)}{2(1-\cos\phi_0)}K(k_0)}\right]. \tag{26}$$

This maximum stress is compare
mation from equation (18) in Fi
for ϕ_0 as large as 45°. In the rem
approximation is adequate for g

Fracture of the lithosphere

Although we have shown that
this is not a criteria for failure
criteria for failure (Nadai 1950),

$\sigma_\theta^2 + ($

where σ_0 is the stress at which t
equations (16) and (17) into equa

$\frac{1}{3}\left(\frac{E\Delta R\phi_0^2}{8R}\right)^2$ [

Failure will occur when the left s
will occur at the position on the sh
This is at $\phi = \phi_0$, i.e. the edge of
fail under tension on the edge if th

In order to determine the me
the principal radii of curvature of
with the result

$R =$

This mean radius of curvature of a
the actual radii of curvature in F

We consider a circular shell w
at the equator, $\gamma = 0$ in equation
function of latitude is then given

Δ

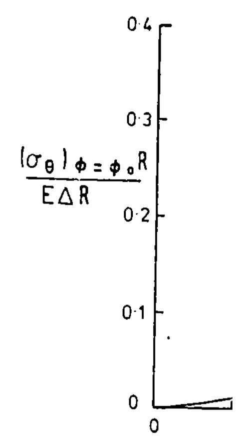

FIG. 4. Dependence of the non
size of the shell using the exact
shell app

This maximum stress is compared with the value given by the shallow shell approximation from equation (18) in Fig. 4. It is seen that the agreement is quite good even for ϕ_0 as large as 45°. In the remainder of this paper we assume that the shallow shell approximation is adequate for geophysical predictions.

Fracture of the lithosphere

Although we have shown that the maximum stress occurs on the edge of the shell, this is not a criteria for failure of the shell. Accepting the Mises–Hencky–Huber criteria for failure (Nadai 1950), the shell will fail when

$$\sigma_\theta^2 + (\sigma_\theta - \sigma_\phi)^2 + \sigma_\phi^2 = 6\sigma_0^2 \tag{27}$$

where σ_0 is the stress at which the material will fail in pure shear. Substitution of equations (16) and (17) into equation (27) gives

$$\tfrac{1}{3}\left(\frac{E\Delta R\phi_0^2}{8R}\right)^2 \left[7\left(\frac{\phi}{\phi_0}\right)^4 - 4\left(\frac{\phi}{\phi_0}\right)^2 + 1\right] = \sigma_0^2. \tag{28}$$

Failure will occur when the left side of equation (28) becomes equal to σ_0^2. Failure will occur at the position on the shell where the left side of equation (28) is a maximum. This is at $\phi = \phi_0$, i.e. the edge of the shell. Therefore we conclude that the shell will fail under tension on the edge if the radius of curvature is increased to a critical value.

In order to determine the membrane stresses in the lithosphere we approximate the principal radii of curvature of the geoid as a function of latitude by a mean value with the result

$$R = a(1 - \varepsilon + 2\varepsilon \sin^2 \gamma). \tag{29}$$

This mean radius of curvature of an equivalent, local spherical shell is compared with the actual radii of curvature in Fig. 1.

We consider a circular shell with a radius of curvature equal to the mean value at the equator, $\gamma = 0$ in equation (29). The change in the radius of curvature as a function of latitude is then given by

$$\frac{\Delta R}{R} = 2\varepsilon \sin^2 \gamma. \tag{30}$$

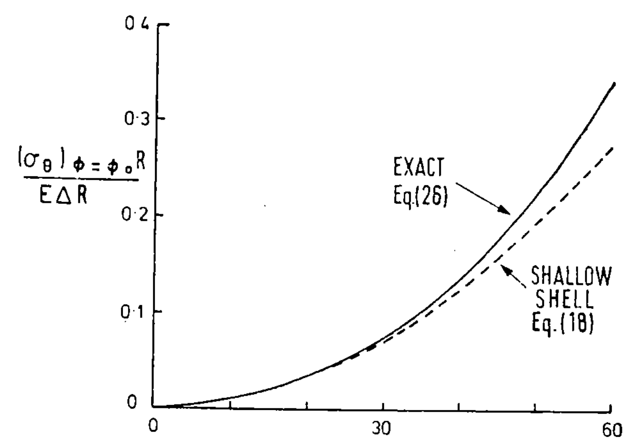

FIG. 4. Dependence of the non-dimensional stress at the edge of the shell on the size of the shell using the exact membrane theory (equation (26)) and the shallow shell approximation (equation (18)).

Substitution of equation (30) into equation (18) gives the maximum tensile stress in the shell

$$(\sigma_\theta)_{\phi=\phi_0} = \tfrac{1}{2} E\, \varepsilon\, \phi_0^{\,2} \sin^2 \gamma. \tag{31}$$

Taking $E = 1{\cdot}72 \times 10^{12}$ dynes/cm^2 (Bullen & Haddon 1967) and $\varepsilon = 0{\cdot}00335$ the stress is given as a function of latitude for several values of ϕ_0 in Fig. 5. We conclude that membrane stresses of the order of several kilobars can be caused by a change in latitude of the lithosphere.

There is considerable uncertainty regarding the stress necessary to cause a fracture of the lithosphere on geological time scales. Stresses on active fault zones are of the order of 0·1 kbar. The strength of mantle rocks in laboratory experiments is of the order of 10 kbar. Membrane stresses of several kilobars may be sufficient to cause propagating tensional fractures in the lithosphere.

Gravity anomalies

The radial surface force on the base of the lithosphere is due to the fluid-like behaviour of the upper mantle. Prior to any deformation the stresses in the surface plate are hydrostatic and the force on the base of the plate is equal to the weight of the plate. If the surface plate is deformed downward into the mantle the surface force, p_r, is positive. If the surface plate is deformed upward the surface force, p_r, is negative. Because of the fluid-like behaviour of the upper mantle it is appropriate to relate the surface force, p_r, to the deviation of the shell from the geoid, d, by the hydrostatic relation

$$p_r = -\rho g d. \tag{32}$$

When the shell is elevated above the geoid, d is positive. As long as $d \ll w$ the above analysis remains valid. It should be emphasized that d is the deviation of the surface from the geoid and w is the deviation of the geoid from a sphere.

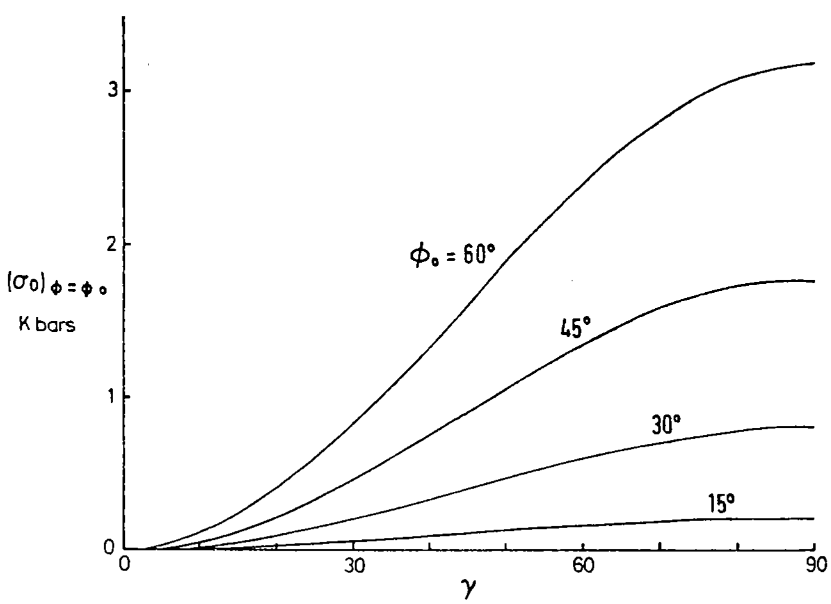

Fig. 5. Maximum stress in several equivalent circular lithospheric plates which are created at the equator and move to a latitude γ.

The gravitational anomaly, Δg, associated with a deviation of the surface from the geoid in the form of a spherical harmonic is

$$\Delta g = \frac{4\pi(n-1)G\rho d}{(2n+1)} \tag{33}$$

where G is the gravitational constant and n is the degree of the spherical harmonic. In order to estimate the gravity anomaly associated with membrane stresses we take the limit $n \to \infty$ in equation (33) with the result

$$\Delta g = 2\pi G\rho d. \tag{34}$$

Taking the value of the surface force given by the shallow shell theory Equation (15), and the change in radius of curvature from equation (30) the predicted gravitational anomaly from equations (32) and (34) is

$$\Delta g = -\frac{2\pi hGE\varepsilon}{ag}\sin^2\gamma(\phi^2 - \tfrac{1}{2}\phi_0{}^2). \tag{35}$$

The maximum value of the gravity anomaly is given by

$$\Delta g = \pm\frac{\pi hGE\varepsilon}{ag}\phi_0{}^2\sin^2\gamma. \tag{36}$$

It is of interest to relate the maximum gravitational anomaly to the maximum tensile stress, substitution of equation (31) into equation (36) gives

$$\Delta g = \pm\frac{2\pi hG}{ag}(\sigma_\theta)_{\phi=\phi_0}. \tag{37}$$

With h = 50 km gravity anomalies of the order of 10 mgal can be caused by membrane stresses of the order of 2 kbar. This is the magnitude of observed gravity anomalies.

Conclusions

The analysis given in this paper shows that the membrane stresses in the lithosphere due to the ellipticity of the Earth are of the order of kilobars. Stresses of this magnitude may be sufficient to fracture the lithosphere. Several authors (Jackson & Wright 1970; Green 1971) have proposed that a propagating fracture may be responsible for the Hawaiian Island chain. There are many examples of such island chains in the oceans and tensional fractures subsequently filled with magmas provide an explanation for their origin. Rift valleys on the continents also appear to be the result of tensional fractures of the lithosphere. Rift valleys display a finite extension which can be explained as the extension required to relieve the membrane stresses in the lithosphere.

In considering the actual distribution of stresses in the lithosphere the analysis given in this paper is only approximately valid. The surface plates are not circular and the geoid is only approximated by a local spherical surface. Also, the lithosphere is formed at different latitudes so that the unstressed radii of curvature will vary over its surface. A large plate could have its radii of curvature increasing in part of the plate and decreasing elsewhere. Fractures could occur in surface plates due to stress concentrations or on zones of previous weakness such as a fault zone. Once started stress concentrations would occur at the tip of the fracture resulting in its propagation.

There are also other sources of stress in the lithosphere which must be added to the membrane stresses to determine the total stress pattern. Thermal stresses may play a significant role. Also, there are the stresses which drive the motion of the plates.

Acknowledgments

The author would like to thank Professor W. S. Hemp for his help in developing the analysis. This work was carried out while the author held a John Simon Guggenheim Memorial Foundation Fellowship.

Department of Geology and Mineralogy,
Parks Road,
*Oxford OX*1 3*PR*

References

Abramowitz, M. & Segun, I. A., 1965. *Handbook of mathematical functions*, Dover Publications, New York.

Bomford, G., 1952. *Geodesy*, Oxford University Press.

Bullen, K. E. & Haddon, R. A. W., 1967. Derivation of an Earth model from free oscillation data, *Proc. natn. Acad. Sci. U.S.A.*, **58,** 846–852.

Green, D. H., 1971. Composition of basaltic magmas as indicators of conditions of origin: application to oceanic volcanism, *Phil. Trans. R. Soc.*, **268A,** 707–725.

Illies, J. H., 1970. Graben tectonics as related to crust–mantle interaction, in *Graben problems*, eds H. Illies and St. Mueller, pp. 4–27, E. Schweizerbartsche Verlagsbuchhandlung, Stuttgart.

Jackson, E. D. & Wright, E. D., 1970, Xenolith's in the Honolulu Volcanic series, Hawaii, *J. Petrology*, **11,** 405–430.

Nadai, A., 1950. *The theory of flow and fracture of solids*, Vol. 1, 2nd ed., pp. 316–327, McGraw–Hill Book Co., New York.

Novozhilov, V. V., 1959. *The theory of thin shells*, pp. 102–117, P. Noordhoff, Groningen, The Netherlands.

Reissner, E., 1946. Stresses and small displacements of shallow spherical shells, *J. Maths. Phys.* **25,** 80–85, and 279–300.

Turcotte, D. L. & Oxburgh, E. R., 1973. Mid-plate tectonics, *Nature*, **244,** 337–339.

Wilkins, G. A., 1965. The system of astronomical constants—Part II, *Q. Jl R. astr. Soc.*, **6,** 70–73.

Geophys. J. R. astr. Soc. (1

Effects of a T
Li

An analytical expre
consisting of a mag
space was first der
Then, with the aid o
sion relating the lat
and the change of t
and time variations
intensity of the ring
Several model ca
physical phenomena

Introduction

Recent studies on the
particles over geocentric d
have confirmed the specul
current in the magnetosphe
magnetosphere is likely to
accumulated evidence ind
ring current and geomagne
The charged particles
many geophysical phenom
system of the Earth and rin
particles. For this reason, i
magnetic lines of force of t
ring current.
For simplicity a system
current in free space, withou
considered. A mathematica
considered was first derived
space and time variations of
of the ring current were exa
derived, were made and t
explored. For convenience,
in the paper without referri

* Received in original form 1